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17MAT11

First Semester B.E. Degree Examination, Aug./Sept. 2020 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Obtain the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. (06 Marks)
- b. Find the angle intersection of the curves $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$. (07 Marks)
- c. Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (07 Marks)

OR

- 2 a. If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
- b. Obtain the pedal equation of the curve,
 $\frac{2a}{r} = (1 + \cos \theta)$. (07 Marks)
- c. Find the radius of curvature for the curve $r^n = a^n \cos n\theta$. (07 Marks)

Module-2

- 3 a. Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x = \frac{\pi}{3}$ upto the fourth degree term. (06 Marks)
- b. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (07 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$. (07 Marks)

OR

- 4 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$. (06 Marks)
- b. Obtain the Maclaurin's expansion of the function $\log(1+x)$ upto the term containing x^4 . (07 Marks)
- c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (07 Marks)

**Module-3**

- 5 a. A particle moves on the curve,
 $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$
where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction of $i - 3j + 2k$. (06 Marks)
- b. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
- c. Prove that $\text{div}(\text{curl } \vec{A}) = 0$ (07 Marks)

OR

- 6 a. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ show that $\vec{F} \cdot \text{curl } \vec{F} = 0$. (06 Marks)
- b. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)
- c. Prove that $\text{curl}(\text{grad } \phi) = 0$. (07 Marks)

Module-4

- 7 a. Evaluate $\int_0^a x\sqrt{ax - x^2} dx$. (06 Marks)
- b. Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$ (07 Marks)
- c. Find the orthogonal trajectories of the family of curves $r = a(1 + \sin \theta)$. (07 Marks)

OR

- 8 a. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$. (06 Marks)
- b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. (07 Marks)
- c. A body in air at 25°C cools from 100°C in 1 minute. Find the temperature of the body at the end of 3 minutes. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix,
$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$
 (06 Marks)
- b. Find the largest Eigen value and the corresponding Eigen vector of the matrix.
$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

by power method taking the initial eigen vector as $[1 \ 0 \ 0]^T$ perform five iterations. (07 Marks)
- c. Show that the transformation,
 $y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Find the inverse transformation. (07 Marks)



OR

- 10 a. Solve the following system of equations by Gauss-Siedel method.
 $20x + y - 2z = 17$
 $3x + 20y - z = -18$
 $2x - 3y + 20z = 25$
Carryout three iterations. (06 Marks)
- b. Reduce the matrix,
 $A = \begin{bmatrix} -1 & 3 \\ -1 & 4 \end{bmatrix}$ to the diagonal form. (07 Marks)
- c. Reduce the following Quadratic form, $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. (07 Marks)
